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All of the content from the author Satya Atluri.
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- Non-Linear Algebraic Equations:

$$\mathbf{F}(\mathbf{X}) = \mathbf{0};$$

unknown vector: $\mathbf{X}$

- Newton's method

$$\mathbf{X}^{k+1} = \mathbf{X}^{k} - \mathbf{B}(\mathbf{X}^{k})^{-1}\mathbf{F}(\mathbf{X}^{k})$$

$$B_{ij} = \frac{\partial F_i}{\partial X_j};$$

Tangent Stiffness.

- Requires an inversion of B in each iteration
- Sensitive to the initial guess of X
- When B is singular, Arc-Length methods are needed

- Liu and Atluri (2008)

$$\dot{\mathbf{X}}(t) = \frac{-\nu}{q(t)}\mathbf{F}(\mathbf{X}) \equiv \mathbf{f}(\mathbf{X}, t) \qquad -\nu \neq 0$$

- Special case: $q(t) = 1 + t$

- **No Inversion of B or H**
- **Not sensitive to initial guess of X**

Ill-Conditioning & Noisy Data:

- Consider the system $\mathbf{AX} = \mathbf{b}$

$\mathbf{A}$: Ill-Conditioned $\quad\quad$ $\mathbf{b}$ is noisy.

Let $\mathbf{A} = \mathbf{U}\, diag\{s_i\}\, \mathbf{V}^T$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{b} = \mathbf{V}\, diag\{s_i^{-1}\}\, \mathbf{U}^T\mathbf{b}$$

- Regularization of Tikhonov [1977]:
- Modify s_i^{-1} for those which are small, compared to others, by: $\omega\left(s_i^2\right)s_i^{-1}$
- Tikhonov Filter $\omega(S) = \dfrac{s}{(s+\alpha)}$

α :Regularization Parameter

- Hansen & O'Leary (1993):

$$\min \Phi(\mathbf{X}) = \min\left\lfloor \|\mathbf{AX} - \mathbf{b}\|^2 + \alpha\|\mathbf{X}\|^2 \right\rfloor$$

$$\left(\mathbf{A}^T\mathbf{A} + \alpha\,\mathbf{I}_n\right)\mathbf{X} - \mathbf{A}^T\mathbf{b} = \mathbf{0}$$

- AE to ODE (Liu & Atluri 2008)

$$\mathbf{A}^T\mathbf{A}\mathbf{X} - \mathbf{A}^T\mathbf{b} = 0$$

$$\mathbf{X}' = \frac{d\mathbf{X}}{d\tau} = -\frac{\nu}{q(\tau)}\left[\mathbf{A}^T\mathbf{A}\mathbf{X} - \mathbf{A}^T\mathbf{b}\right]$$

Let $t = \int_0^\tau \frac{\nu}{q(\xi)} d\xi$

$$\dot{\mathbf{X}} = \frac{d\mathbf{X}}{dt} = -\mathbf{A}^T\mathbf{A}\mathbf{X} + \mathbf{A}^T\mathbf{b}$$

$$\mathbf{X}(t) = \mathbf{V}\ diag\left\{\omega(s_i)s_i^{-1}\right\}\mathbf{U}^T\mathbf{b}$$

$$\omega(s) = 1 - e^{-ts}$$

Let $q(\tau) = (1+\tau)^{\gamma} \quad 0 \le \gamma < 1$

$$t = \nu \ln(1+\tau) \qquad \gamma = 1$$

$$= \frac{\nu}{1-\gamma}\left[(1+\tau)^{1-\gamma} - 1\right] \qquad 0 \le \gamma < 1$$

$$\omega(s) = 1 - \frac{1}{(1+\tau)^{\nu s}} \qquad \gamma = 1$$

$$= 1 - \exp\left(\frac{-s}{1-\gamma}\left[(1+\tau)^{1-\gamma} - 1\right]\right) \quad 0 \le \gamma < 1$$

$$\dot{\mathbf{X}} = \frac{d\mathbf{X}}{dt} = -\left[\mathbf{A}^T\mathbf{A} + \alpha\,\mathbf{I}_n\mathbf{X}\right] + \mathbf{A}^T\mathbf{b}$$

$$\omega(s) = \frac{1 - e^{-t(s+\alpha)}}{(s+\alpha)}\,s$$

Filter in the current FTIM

Solved by Group Preserving Scheme

- Ill-posed Linear Problems, Noisy Data & Filtering.

- Differentiation of Noisy-Data

a Crack-length.

$\frac{da}{dN}$: rate of Crack-growth , per fatigue cycle, N

$$\left(\frac{da}{dN}\right) = C(a)^n = d(\Delta K)^n$$

How do you find $\left(\frac{da}{dN}\right)$ from the

a vs N data, which may be noisy?

- Fatigue Crack-Growth: $\left(\frac{da}{dN}\right)$
- We start with experimentally determined $a \; vs \; N$ curve with discrete data:

$$a(N_i) = a_i$$

- Consider a Noise

$$\hat{a}(N_i) = a(N_i) + \sigma R(N_i)$$

$$R(N_i) = Random$$

- Consider the Laplace transform of $\left(\frac{da}{dN}\right)$

- Finding da/dN from Noisy Data for a versus N

$a \; and \; (da/dN)$ are assumed to be defined in the interval $[0, N]$

$$\int_0^N e^{-sN} \frac{da}{dN} dN = s\int_0^N e^{-sN} a(N) dN - a(0)$$

$$\left(\frac{\Delta N}{2}\right) K_{i,1}\left[\frac{da}{dN}(N_1)\right] + \Delta N \sum_{j=2}^{m} \left(\frac{da}{dN}\right)_{ij} + \left(\frac{\Delta N}{2}\right) K_{i,m+1}\left[\frac{da}{dN}\right](N_{m+1}) = h[N_i]$$

$$N_i = (i-1)\Delta N = (i-1)\frac{N}{m} \qquad s_i = (i-1)\Delta s$$

$$K_{ij} = \exp\left[-s_i N_j\right]$$

$$h(N_i) = s_i[\frac{\Delta N}{2} K_{i,1} \hat{a}(N_i)$$

$$+\Delta N \sum_{j=2}^{m} K_{i,j} \hat{a}(N_j)$$

$$+\frac{\Delta N}{2} K_{i,m+1} \hat{a}(N_{m+1}) - \hat{a}(N_i)]$$

$$[A]\left\{\frac{da(N_i)}{dN}\right\} = h(N_i)$$

$$= s_i[\frac{\Delta N}{2} K_{i,1} \hat{a}(N_i)$$

$$+ \Delta N \sum_{j=2}^{m} K_{i,j} \hat{a}(N_j)$$

$$+ \frac{\Delta N}{2} K_{i,m+1} \hat{a}(N_{m+1}) - \hat{a}(N_1)]$$

$$\hat{a}(N_i) = given = a(N_i) + \sigma R(N_i)$$

$$\mathbf{AX} = \mathbf{b} \qquad \mathbf{X}' = \frac{d\mathbf{X}}{d\tau} = -\frac{\nu}{1+\tau}\left[\mathbf{A}^T\mathbf{AX} - \mathbf{A}^T\mathbf{b}\right]$$

Solved by Group Preserving Scheme

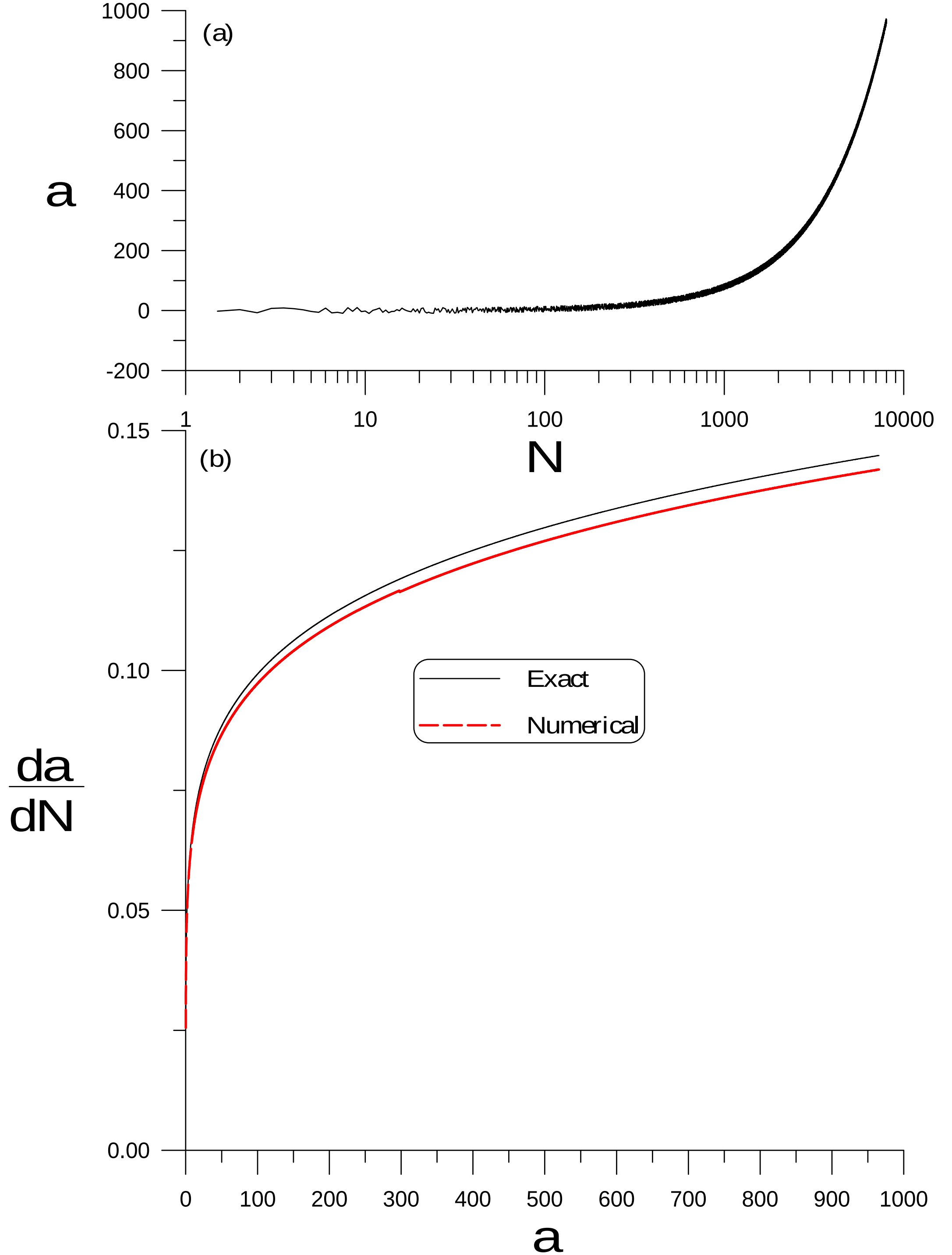
- Example:

$$\Delta\tau = 10^{-4}$$

$$m = 1000$$

$$\nu = 0.05$$

$$r = 1$$



Comparison of the numerical and exact solutions for obtaining d*a*/d*N* versus *a* from the measured noisy data for crack-length *a* versus the number of cycles, *N*.

- Inverse Laplace Transform

$$\int_0^{\infty} e^{-st} f(t)dt = F(s)$$

Given $F(s)$,find $f(t)$

Let $f(t) = \sum_{j=1}^{n} C_j \left(\frac{t}{T_0}\right)^{j-1}$

$$C_1 + \sum_{j=2}^{n} \frac{(j-1)!C_j}{(T_0 s)^{j-1}} = sF(s)$$