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2009 : October 2009 - Emerging Research Fronts : Ralf Metzler & Joseph Klafter on Anomalous Transport by Fractional Dynamics

EMERGING RESEARCH FRONTS - 2009

October 2009

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Ralf Metzler & Joseph Klafter talk with *ScienceWatch.com* and answer a few questions about this month's Emerging Research Front Paper in the field of Mathematics.

Article: The restaurant at the end of the random walk: recent developments in the description of anomalous transport by fractional dynamics
Authors: Metzler, R;Klafter, J
Journal: J PHYS-A-MATH GEN, 37 (31): R161-R208 AUG 6 2004
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SW: Why do you think your paper is highly cited?

The paper represents an up-to-date introduction to the mathematics of diffusion in complex systems involving Lévy stable laws which, in turn, are connected to the generalized central limit theorem. The paper summarizes a large body of experimental findings of anomalous diffusion phenomena, and is therefore useful both for theorists and experimentalists alike.

The paper is a follow-up on our first review article on the fractional Fokker-Planck equation: Metzler, R; Klafter, J, "The random walk's guide to anomalous diffusion: a fractional dynamics approach," *Phys Rep-Rev Sect Phys Lett* 339:1-77, DEC 2000. The article was an *Essential Science IndicatorsSM* New Hot Paper selection in May 2003.

It collects the recent developments in the fast growing field of anomalous transport processes, covering subdiffusion and emphasizing superdiffusion phenomena. The collected evidence demonstrates that anomalous diffusion is ubiquitous and reaches into a large variety of fields, such as physics, chemistry, geophysics, astrophysics, but also the biological, financial, and sociological fields.

SW: Does it describe a new discovery, methodology, or synthesis of knowledge?

The fractional Fokker-Planck equation in the form presented in R. Metzler, *et al.*, "Anomalous diffusion and relaxation close to thermal equilibrium: A fractional Fokker-Planck equation approach," *Phys. Rev. Lett*. 82: 3563-67, 1999; and in R. Metzler, *et al.*, "Deriving fractional Fokker-Planck equations from a generalized master equation," *Europhys. Lett*. 46: 431-36, 1999; is a general framework for subdiffusion and superdiffusion in external force fields.

Much as the regular Fokker-Planck equation, it provides a very general description for random renewal processes with inherent, long-tailed memory. It is connected with the Mittag-Leffler relaxation pattern identified in an increasing number of complex systems, ranging from nano and microscales to geophysics scales.

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We also report on the space-fractional Fokker-Planck equation that describes random walk processes in external force fields governed by a long-tailed distribution of jump lengths with diverging variance. Such Lévy flights arise naturally in jump models on annealed polymer chains with intersegmental jumps and serve as models for large amplitude noise.

In this paper we especially present recent findings on how to tame Lévy flights: In steeper than harmonic potentials the resulting process possesses finite variance and other interesting properties.

Moreover, in this paper we covered recent results on the first passage behavior in both sub- and superdiffusive regimes. This paper is therefore a guidebook for scientists working on transport phenomena within complex systems.

SW: Would you summarize the significance of your paper in layman's terms?

The classical drunken sailor's walk—after Karl Pearson (1857–1936), English mathematician, biometrician, and statistician—describes the trajectory of a walker moving in a random direction with each step. The steps occur at regular time intervals. Such a process leads to normal diffusion in the continuum limit, governed by the famed Gaussian form of the probability density to find the walker at some position at a given time.

The situation changes drastically when we introduce variable pauses between successive steps. For instance, the already drunken sailor visits additional pubs on his way. Assume that some of these pubs have such a good beer that the sailor spends exceedingly long times in them. Then he most likely will not make it back to his ship before its departure.

In nature, similar phenomena may occur. Imagine a tracer chemical, e.g., chloride, dissolved in the rainwater coming down on a catchment. The tracer may get trapped inside channels off the main water artery. If these side channels are very long, the transport of the tracer is governed by the power-law return to the main artery. This causes a wide, power-law distribution of waiting times with a diverging mean waiting time. If there is an additional external force acting on the particle along the main artery, its motion is described by the time-fractional Fokker-Planck equation.

Lévy flights, i.e., random walk processes with power-law distribution of jump lengths but typical waiting time between jumps, are similarly described as a variant of the fractional Fokker-Planck equation.

SW: How did you become involved in this research and were any particular problems encountered along the way?

RM became involved in fractional differential equations during his thesis work at the University of Ulm. When he came to Tel Aviv University to join JK, who had an interest in anomalous diffusion and Lévy flights, we realized that these equations are ideally suited to describe continuous time random walk processes with power-law waiting time or jump length distributions, in the presence of external force fields.

When one needs to solve a regular diffusion process in a force field, mostly one would start with the Fokker-Planck equation. The fractional Fokker-Planck equation is the starting point at exactly the same level for particles performing anomalous diffusion. A challenge is how to phrase such an equation if we are dealing with spatiotemporally coupled Lévy walks.

SW: Where do you see your research leading in the future?

Currently, scientists in the field are dealing with the subtleties of such anomalous diffusion properties when we calculate time averages instead of the common ensemble averages. In fact, for a subdiffusion process described by the fractional Fokker-Planck equation, ergodicity is broken in the sense that the ensemble and time averages are no longer the same, caused by the ageing properties of the underlying process. This also leads to the question how one may better analyze measured single-particle trajectories.

Another open question is that of molecular crowding in biological cells: what mechanism exactly causes the observed subdiffusion?

SW: Do you foresee any social or political implications for your research?

Realizing that diverging time scales govern certain processes should eventually change the mindset within various fields. Prime examples are that of groundwater studies and their related implications. Thus, spillage of chemicals that eventually get into the groundwater will pose a long-term challenge, as the chemicals are not washed out quickly, but a good portion stays in the aquifer for considerable periods of time. Over decades, this may compromise the water quality.

Lévy flights and walks have found their way to describe social phenomena such as human mobility and the spread of epidemics. For instance, the high connectivity of air traffic makes it possible to spread diseases over continents within a time scale of only several days.

In the future, a better understanding of these related dynamics will help to develop better measures against the spreading of diseases. Similarly, better quantitative understanding of human mobility behavior will make it possible to improve the design of the infrastructure of the future.

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